Effects of Velocity Slip and Viscosity Variation in Slider Bearings with Two Layer Lubricant

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Abstract:A theoretical study of slider bearing considering viscosity variation across the film and slip of the bearing surface with thermal effect is presented. A generalized Reynolds equation for the lubrication of two layers is derived. It is applied to study these effects in slider bearing by taking the fluid film into three zones. Expressions for pressure, load capacity and force of friction are studied by evaluating them numerically for various parameters.

Index Terms: viscosity variation, velocity-slip, slider bearings, load capacity, coefficient of friction.

1. INTRODUCTION

In general most of the lubricated systems can be considered to consist of moving or stationary surfaces with a thin film of an external material between them. In most mechanical systems where relative motion occurs between two parts lubricants are introduced to reduce friction and wear. The geometry of the contacting elements determines the shape of the lubricant film. Various researchers have considered different configurations of the lubricating film in the clearance zone in their analysis. Traditionally hydrodynamic equations have been used to model slider bearing problems. Boundary conditions for hydrodynamic was widely discussed during 19th century.

In 1867 Maxwell applied his kinetic theory [1] to study of the slip length associated with fluid close to a solid surface and discover the velocity slip effect near a moving wall. Qvale and Wiltshire [2] studied a concept pf multiple layers lubrication is proposed here so that the effect of viscosity variation across the film near the solid surfaces can be taken into account. Ramnaiah, G. [3] studied slider bearings lubricated by fluids with couple stress. Bujurke, N.M [4] investigated slider bearings lubricated by a second order fluid with reference to synovial joints. Yurusoy [5] obtained a perturbation solution for pressure distribution in a slider bearing with a Powel-Eyring fluid as lubricant. Andhariah [6] studied the effects of surface roughness on hydrodynamic lubrication of slider bearings. B.V.R. Kumar [7] studied the performance of a Slider bearing with heat conduction to the pad. Bayrakceken [8] carried out a comparative study of inclined and parabolic slider bearings using a non-Newtonian fluid in the clearance zone and developed closed form expressions for the performance metrics. Ozalap [9] studied optimum surface profile design and performance evaluation of inclined slider bearings. Gupta, J. L. [10] analyzed the behavior of a hydrodynamic squeeze film between a non-rotating spherical surface and a hemispherical bearing under a steady load. Naduvinamani, N. B et.al [11] studied surface roughness effects on curved pivoted slider bearings with couple stress fluid. Raghavendra Rao [12] studied the effects of velocity slip and viscosity variation for lubrication of Roller bearings. Das, N. C [13] studied optimum load bearing capacity for slider bearings lubricated with couple stress fluids in magnetic field. . Bujurke, N.M [14] studied a porous slider beraing with couple stress fluid. Wen-Ming Zhan [15] developed a mathematical model for the slip flow in an ultra-thin-film gas-lubricated microbearing with surface roughness effect. Shah [16] computed values for the bearing characteristics of a secant shaped slider bearing using a magnetic fluid as lubricant.

The present paper focuses on the effects of velocity slip and viscosity variation across the film at the surfaces of slider bearings with thermal effects.

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2. BASIC EQUATIONS

A generalized form of Reynolds equation for compressible fluid film lubrication considering velocity slip at the bearing surface [11] is given by

$$\frac{\partial}{\partial x}\left[(F_2+G_1)\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[(F_2{}^1+G_1{}^1)\frac{\partial p}{\partial y}\right]=H_2\left[\frac{\partial}{\partial z}(\rho u)_2+\frac{\partial}{\partial y}(\rho v)_2\right]-H_1\left[\frac{\partial}{\partial x}(\rho u)_1+\frac{\partial}{\partial y}(\rho v)_1\right]$$
$$-\frac{\partial}{\partial x}\left[\frac{(U_2-U_1)(F_3+G_2)}{F_0}+U_1G_3\right]-\frac{\partial}{\partial y}\left[\frac{(V_2-V_1)(F_3+G_2{}^1)}{F_0{}^1}+V_1G_3\right]+\int_{H_1}^{H_2}\frac{\partial\rho}{\partial t}dz+[\rho w]_{H_1}^{H_2} \qquad (1)$$

where $F_2=\int_{H}^{H_2}\frac{\rho z}{\eta}\left(z-\frac{F_1}{F_0}\right)dz$

$$F_2{}^1=\int_{H}^{H_2}\frac{\rho z}{\eta}\left(z-\frac{F_1{}^1}{F_0{}^1}\right)dz$$

$$F_3=\int_{H}^{H_2}\frac{\rho z}{\eta}dz$$

$$G_1=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\left\{\left(\alpha_1H_1+\int_{H_1}^{z}\frac{z\,dz}{\eta}\right)-\frac{F_1}{F_0}\left(\alpha_1+\int_{H_1}^{z}\frac{dz}{\eta}\right)\right\}\right]dz$$

$$G_1{}^1=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\left\{\left(\beta_1H_1+\int_{H_1}^{z}\frac{z\,dz}{\eta}\right)-\frac{F_1{}^1}{F_0{}^1}\left(\beta_1+\int_{H_1}^{z}\frac{dz}{\eta}\right)\right\}\right]dz$$

$$G_2=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\left(\alpha_1+\int_{H_1}^{z}\frac{dz}{\eta}\right)\right]dz$$

$$G_2{}^1=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\left(\beta_1+\int_{H_1}^{z}\frac{dz}{\eta}\right)\right]dz$$

$$G_3=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\right]dz$$

The two sets of functions F and G depends upon the variation of fluid properties along as well as across the film and on the velocity-slip conditions at the surfaces i.e.,

$$(\lambda)_1=(\lambda)_2=(\delta)_1=(\delta)_2=0$$
$$\alpha_1=\alpha_2=\beta_1=\beta_2=0$$

In the case of incompressible lubricant all G- functions vanish and generalized Reynolds equation with slip (1) takes a simple form

$$\frac{\partial}{\partial x}\left[F_2\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[F_2{}^1\frac{\partial p}{\partial y}\right]=H_2\left[\frac{\partial}{\partial z}(u)_2+\frac{\partial}{\partial y}(v)_2\right]-H_1\left[\frac{\partial}{\partial x}(u)_1+\frac{\partial}{\partial y}(v)_1\right]$$
$$-\frac{\partial}{\partial x}\left[\frac{(U_2-U_1)F_3}{F_0}\right]-\frac{\partial}{\partial y}\left[\frac{(V_2-V_1)F_3}{F_0{}^1}\right]+[w]_{H_1}^{H_2} \qquad (2)$$

where $F_0=\alpha_1+\alpha_2+\int_{H_1}^{H_2}\frac{dz}{\eta}$

$$F_0{}^1=\beta_1+\beta_2+\int_{H_1}^{H_2}\frac{dz}{\eta}$$

$$F_1=\alpha_1H_1+\alpha_2H_2+\int_{H_1}^{H_2}\frac{z\,dz}{\eta}$$

$$F_1{}^1=\beta_1H_1+\beta_2H_2+\int_{H_1}^{H_2}\frac{z\,dz}{\eta} \qquad (3)$$

$$F_2=\int_{H}^{H_2}\frac{z}{\eta}\left(z-\frac{F_1}{F_0}\right)dz$$

$$F_2{}^1=\int_{H}^{H_2}\frac{z}{\eta}\left(z-\frac{F_1{}^1}{F_0{}^1}\right)dz$$

$$F_3=\int_{H}^{H_2}\frac{z}{\eta}dz$$

Let us consider the flow of three fluid layers in the case of a slider bearing whose configuration is shown in the following Figure

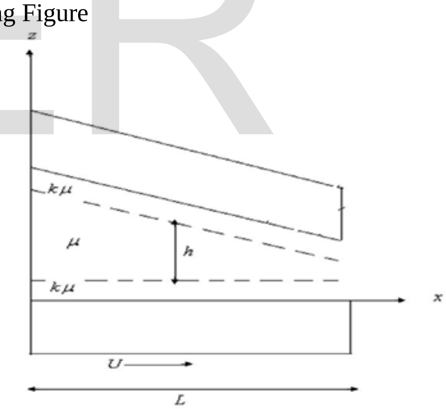

Figure: Slider bearing with three layers of Lubricant

Considering also the effects of slip at the surface is

$$\lambda=\frac{\eta_1}{\beta}$$

where η_1 is the viscosity of the lubricant at the surface and β is the coefficient of sliding friction at the surface. It is noted here, that as β increases λ decreases and β tends to infinity in the case of no-slip at the surface. Now keeping in view the physical situation represented in above Figure and considering the following

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$$U_1 = U, \qquad U_2 = V_1 = V_2 = 0$$

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \frac{1}{\beta} \quad (4)$$

$\rho =$ constant, $\eta_1, \eta_2 =$ constant

$H_1 = 0, \quad H_2 = h + H$

The Reynolds equation for multiple viscous incompressible layers lubrication is derived from equation (2) as follows

$$\frac{\partial}{\partial x}\left[F_2 \frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_2 \frac{\partial p}{\partial y}\right] = (h+H)\left[\frac{\partial}{\partial x}(u)_2 + \frac{\partial}{\partial y}(v)_2\right] + U\frac{\partial}{\partial x}\left(\frac{F_3}{F_0}\right) + [w]_0^{h+H} \quad (5)$$

where $F_0 = \frac{h}{\eta_2} + \frac{H}{\eta_1} + \frac{2}{\beta}$

$$F_2 = \frac{h^3}{12\eta_2} + \frac{H^3 + 3H^2 h + 3Hh^2}{12\eta_1}$$

$$F_3 = \left(\frac{h+H}{2}\right)\left(\frac{h}{\eta_2} + \frac{H}{\eta_1}\right)$$

$$(u)_2 = -\frac{1}{\beta}\left(\frac{h+H}{2}\right)\frac{\partial p}{\partial x} + \frac{U}{\beta F_0} \quad (6)$$

$$(v)_2 = -\frac{1}{\beta}\left(\frac{h+H}{2}\right)\frac{\partial p}{\partial y}$$

$$[w]_0^{h+H} = (u)_2 \frac{\partial}{\partial x}(h+H) + (v)_2 \frac{\partial}{\partial y}(h+H) - V$$

Simplifying further equation (5) by using eq (6), we get,

$$\frac{\partial}{\partial x}\left[F_4 \frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_4 \frac{\partial p}{\partial y}\right] = \frac{U}{2}\frac{\partial}{\partial x}(h+H) - V \quad (7)$$

where

$$F_4 = \frac{h^3}{12\eta_2} + \frac{H^3 + 3H^2 h + 3Hh^2}{12\eta_1} + \frac{(h+H)^2}{2\beta}$$

(8)

3. METHOD OF SOLUTION

The one-dimensional form of equation governing the pressure in the fluid film, by taking $\eta_1 = k\rho, \ \eta_2 = \mu$ is

$$\frac{d}{dx}\left[F_4 \frac{dp}{dx}\right] = \frac{U}{2}\frac{d}{dx}(h+H) - V \quad (9)$$

Where, $F_4 = \frac{h^3}{12\mu}\left[\frac{\left(1-\frac{a}{h}\right)^3 (k-1)+1}{k} + \frac{6\mu}{h\beta}\right]$ (10)

Here β represents the slip parameter, k the viscous layer parameter, a the thickness of the peripheral layer, h the total film thickness of the lubricant and μ the viscosity of the middle layer.

Now it is assumed that the Newtonian viscosity is varying along the fluid thickness h, therefore

$$\mu = \mu_1 \left(\frac{h}{h_1}\right)^q \quad (11)$$

where μ_1 is the inlet viscosity.

The parameter $q\,(0 \le q \le 1)$ depends on the particular lubricant used for perfect Newtonian fluids *q=0*, whereas for perfect gases *q*=1

$$\frac{d}{dx}\left[F_4 \frac{dp}{dx}\right] = \frac{U}{2}\frac{d}{dx}(h+H) - V \quad (12)$$

Where,

$$F_4 = \frac{h^3}{12\mu_1}\left(\frac{h_1}{h}\right)^q \left[\frac{\left(1-\frac{a}{h}\right)^3 (k-1)+1}{k} + \frac{6\mu_1 h^q}{h h_1^q \beta}\right] \quad (13)$$

The film thickness in this case is given by

$$h = h_1 - (h_1 - h_0)\frac{x}{L}$$

Integrating the above equation (12) with the boundary condition

$$\frac{dp}{dx} = 0 \qquad at \ x = x_0, h = h_0$$

We get,

$$\frac{dp}{dx} = \frac{\frac{U}{2}(h-h_0)+V(x_0-x)}{F_4} \tag{14}$$

By following the usual procedure, the load capacity and force of friction can be written as

$$W = \int_0^L \left(-x\frac{dp}{dx}\right)dx$$

$$W = -\int_0^L x\left[\frac{\frac{U}{2}(h-h_0)+V(x_0-x)}{F_4}\right]dx \tag{15}$$

$$f = \int_0^L \eta\left(\frac{\partial u}{\partial z}\right)_{z=0} dx$$

$$f = \int_0^L \left[\frac{U}{F_0} - \frac{h}{2}\,\frac{\frac{U}{2}(h-h_0)+V(x_0-x)}{F_4}\right]dx \tag{16}$$

Where $F_0 = \dfrac{h-2a}{\mu_1\left(\frac{h}{h_1}\right)^q} + \dfrac{2a}{k\mu_1\left(\frac{h}{h_1}\right)^q} + \dfrac{2}{\beta}$

$$F_4 = \frac{h^3}{12\mu}\left[\frac{\left(1-\frac{a}{h}\right)^3(k-1)+1}{k} + \frac{6\mu}{h\beta}\right]$$

Introducing non dimensional parameters

$$\bar{x} = \frac{x}{L}, \qquad \bar{x}_0 = \frac{x_0}{L}, \qquad \bar{a} = \frac{a}{h_1},$$

$$\bar{\beta} = \frac{h_1\beta}{\mu_1}, \qquad \bar{v} = \frac{VL}{Uh_1}$$

$$\bar{h} = 1-(1-\alpha)\bar{x} \qquad \text{where } \alpha = \frac{h_0}{h_1}$$

The non dimensional load capacity is given by

$$\overline{W} = \frac{Wh_1^{\,2}}{6\mu_1 UL^2}$$

$$\overline{W} = -\int_0^1 \bar{x}\left[\frac{(\bar{h}-\alpha)+2\bar{v}(\bar{x}_0-\bar{x})}{\overline{F}_4}\right]d\bar{x} \tag{17}$$

The non dimensional force of friction is given by

$$\bar{f} = \frac{fh_1}{\mu_1 UL}$$

$$\bar{f} = \int_0^1 \left[\frac{1}{\overline{F}_0} - \frac{3\bar{h}\left[(\bar{h}-\alpha)+2\bar{v}(\bar{x}_0-\bar{x})\right]}{\overline{F}_4}\right]d\bar{x} \tag{18}$$

where

$$\overline{F}_4 = (\bar{h})^{3-q}\left[\frac{\left(1-\frac{\bar{a}}{\bar{h}}\right)^3(k-1)+1}{k} + \frac{6}{(\bar{h})^{1-q}\bar{\beta}}\right]$$

and $\overline{F}_0 = \dfrac{\bar{h}-2\bar{a}}{\bar{h}^q} + \dfrac{2\bar{a}}{k\bar{h}^q} + \dfrac{2}{\bar{\beta}}$

The bearing characteristics i.e., load carrying capacity and force of friction can be evaluated numerically as they cannot be obtained by direct integration.

4. RESULTS & DISCUSSIONS

The parameters considered here are $\bar{\beta}, \bar{a}, q$ and *k*. *k* represents the ratio of the viscosities of the peripheral layer to the middle layer and $\bar{a}$, the thickness of the peripheral layer. $\bar{\beta}$ represents the non-dimensional slip parameter. Low values of $\bar{\beta}$ indicates high slip at the surface and as $\bar{\beta}$ increases the slip decreases and it tends to zero for high values of $\bar{\beta}$. Thus an increase in $\bar{\beta}$ indicates decreasing the slip at the surfaces.

Fig.1 presents the dimensionless load carrying capacity $\overline{W}$ with the thickness of the peripheral layer $\bar{a}$ for various values of *k*, it is seen from the graph that for *k=1* it is parallel to x-axis. i.e., when the peripheral layer viscosity is same as the middle layer viscosity, the effect of increase in the peripheral layer is nil. When *k<1* the load capacity decreases as $\bar{a}$ increases, i.e., when then peripheral layer viscosity is less than the middle layer viscosity, the load capacity decreases. When *k>1* the load capacity increases as $\bar{a}$ increases, i.e., when the peripheral layer viscosity is higher than the middle layer viscosity, the load capacity increases.

The variation of the dimensionless load carrying capacity $\overline{W}$ with thermal factor q for various $\overline{\beta}$ is depicted in Fig.2. It is observed that $\overline{W}$ decreases for decreasing values of $\overline{\beta}$. The load capacity decreases as thermal factor q increases.

In Fig.3 the load carrying capacity $\overline{W}$ is plotted against $\overline{\beta}$ for various values of k, treating $\overline{a}$ as constant. It is observed that $\overline{W}$ increases rapidly as the slip parameter $\overline{\beta}$ increases up to certain level and the load capacity decreases as the velocity slip increases.

The frictional parameter $\overline{f}$ is plotted against q, the thermal factor in Fig. 4 for different values of $\overline{\beta}$. It is seen from the Figure, that the friction parameter $\overline{f}$ decreases as the thermal factor increases for all values of $\overline{\beta}$ and $\overline{f}$ increases as $\overline{\beta}$ increases.

In Fig.5 the frictional parameter $\overline{f}$ is plotted against $\overline{\beta}$ for various values of k, treating $\overline{a}$ as constant. It is observed that $\overline{f}$ decreases as the slip parameter $\overline{\beta}$ increases. i.e., the frictional parameter $\overline{f}$ decreases as the velocity slip increases.

FIGURES

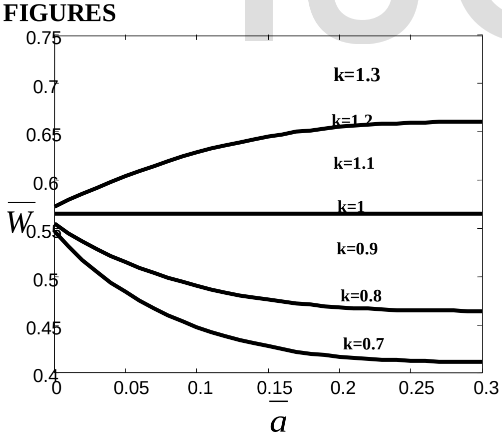

Fig.1 : Variation of $\overline{W}$ with $\overline{a}$ for various k

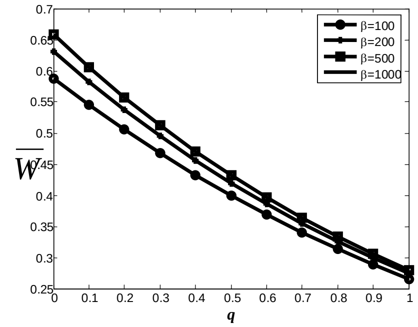

Fig. 2 : Variation of $\overline{W}$ with q for various $\overline{\beta}$

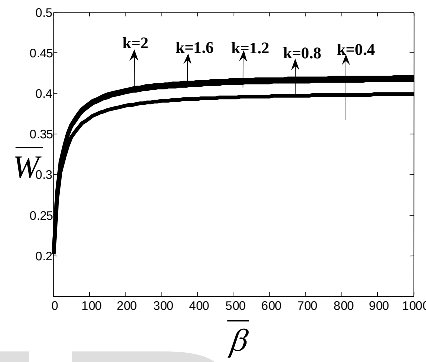

Fig. 3 : Variation of $\overline{W}$ with $\overline{\beta}$ for various k

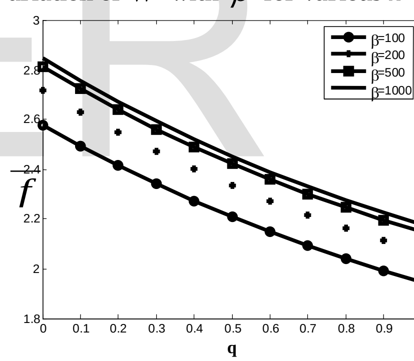

Fig. 4 : Variation of $\overline{f}$ with q for various $\overline{\beta}$

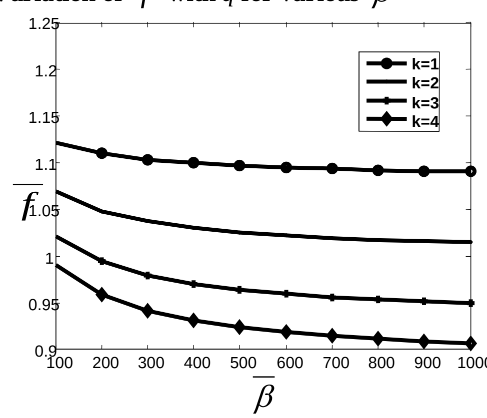

Fig. 5 : Variation of $\overline{f}$ with $\overline{\beta}$ for various k

5. CONCLUSION

In the present paper a generalized form of Reynolds equation for multiple incompressible layers by considering the effects of slip at the surface and the variation of fluid properties across the film and it is applied to study the effects of velocity-slip and viscosity variation on the hydrodynamic lubrication of slider bearings is presented. It is found that numerically, the load carrying capacity increases due to the high viscous layer near the surface and this increase is more as its thickness increase. The slip parameter decreases load capacity. The effect of these parameters on frictional force is also studied numerically.

Nomenclature:

a Thickness of the peripheral layer
f Frictional force
$\overline{f}$ Non dimensional frictional parameter
h Total film thickness
h_0 Film thickness where maximum pressure occurs
h_1 Inlet film thickness
H_1, H_2 Distance between the surfaces
k Ratio of the peripheral layer
L Length of the bearing
p Hydrodynamic pressure
q Thermal factor
u, v, w Velocity components of the film in x, y and z directions
U_1, U_2 Velocities of the surfaces at $z = H_1$ and $z = H_2$ in x direction
U Rolling velocity of the cylinders
V_1, V_2 Velocities of the surfaces at $z = H_1$ and $z = H_2$ in y direction
V Squeeze velocity
W Load capacity
$\overline{W}$ Non dimensional load capacity
x, y, z Rectangular coordinates
ρ Density of the lubricant
η, η_1, η_2 Viscosities of the lubricant
μ Viscosity of the base lubricant
μ_1 Inlet viscosity
β Slip parameter

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